Hyper-elastic constitutive equations of conjugate stresses and strain tensors for the Seth–Hill strain measures

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Abstract

The use of hypo-elastic constitutive equations for large strains in nonlinear finite element applications usually requires special considerations. For example, the strain does not tend to zero upon unloading in some elastic loading–unloading closed cycles. Furthermore, these equations are based on objective material time rate tensors, which require incrementally objective algorithms for numerical applications and integration. Hyper-elastic constitutive equations on the other hand do not require such considerations. However, their behaviour for large elastic strains is important and may differ in tension and compression. In the present work, Hyper-elastic constitutive equations for the Seth–Hill strains and their conjugate stresses are explored as a natural generalisation of Hook’s law for finite elastic deformations. Based on the uniaxial and simple shear tests, the response of the material for different constitutive equations is examined. Together with an objective rate model, the effect of different constitutive laws on Cauchy stress components is compared. It is shown that the constitutive equation based on logarithmic strain and its conjugate stress gives results closer to that of the rate model. In addition, the use of Biot stress–strain pairs for a bar element results in an elastic spring which obeys the Hook’s law even for large deformations and has the same behaviour in both tension and compression. The effect of the constitutive equation on the volume change of the material has also been considered here.

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1. Introduction

The concept of energy conjugacy first presented by Hill [1] states that a stress measure $T$ is said to be conjugate to a strain measure $E$ if $T : E$ represents power or rate of change of internal energy per unit reference volume, $\dot{w}$. That is

$$\dot{w} = III \sigma : D = T : \dot{E}$$

(1)

where $\sigma$ and $D$ are the Cauchy stress and strain rate tensors, respectively, $III = \det(U)$ is the third invariant of the right stretch tensor $U$ and $(')$ is material time derivative operator. Spectral decomposition theorem allows recasting of $U$ as:

$$U = \sum_{i} \lambda_i N_i \otimes N_i,$$

(2)

where $\{\lambda_i\}$ and $\{N_i\}$ are the principal stretches and corresponding orthonormal eigenvectors of $U$, respectively.

The Seth–Hill class of strain measure tensors $E^{(m)}$ indexed by superscript $m$ are defined as:

$$E^{(m)} = \frac{1}{m} \sum_{i} (\lambda^m_i - 1) N_i \otimes N_i = \frac{1}{m} (U^m - I); \quad \text{if} \ m \neq 0$$

$$E^{(0)} = \sum_{i} \ln(\lambda_i) N_i \otimes N_i = \ln U$$

(3)

where $I$ is the identity tensor. Guo and Man [2] derived explicit tensorial formulations for conjugate stress $T^{(m)}$ for $|m| \geq 3$, while earlier, the stress measure conjugate to logarithmic strain tensor $\ln U$ had been derived by Hoger [3].

Also, following the Hill’s principal axis method and energy conjugacy notion, a relation between the components of two Seth–Hill conjugate stress tensors in the principal axes $\{N_i\}$ is derived [10]:

$$T_{ij}^{(n)} = \frac{n}{m} T_{ij}^{(m)} \frac{\sum_{r=1}^{m} \lambda_{i}^{m-r} \lambda_{j}^{r-1}} {\sum_{r=1}^{n} \lambda_{i}^{n-r} \lambda_{j}^{r-1}}$$

(4)

where $T^{(n)}$ and $T^{(m)}$ are stress measures conjugate to $E^{(n)}$ and $E^{(m)}$, respectively, and

$$T^{(n)} = \sum_{i,j} T_{ij}^{(n)} N_i \otimes N_j$$

(5)

According to definition, a material is called hyper-elastic if there is a potential function $w$ such that $T^{(2)} = \partial w / \partial E^{(2)}$. In this work, the hyper-elastic constitutive equation

$$T^{(n)} = 2\mu E^{(n)} + \lambda \text{tr}(E^{(n)}) I$$

(6)
is investigated for finite elastic deformations for some positive and negative $n$ values, where $\mu$ and $\lambda$ are material elastic constants and $\text{tr}(\cdot)$ stands for trace operator. It is shown that for different $n$s, Eq. (6) results in different material behaviour for large elastic strains which are compared with an objective rate model and the variation of Cauchy stress components due to different constitutive models are examined.

For a deforming body, with $F$ denoting the deformation gradient at a material point and $\det(F) > 0$, the polar decomposition theorem states that $F$ may be uniquely decomposed as:

$$F = RU = VR$$

(7)

where $U$ and $V$ are the right and left stretch tensors, respectively and are both positive definite symmetric tensors and $R$ is a rotation.

The eigenvalues of $U$ and $V$, called principal stretches, are denoted by $\lambda_1$, $\lambda_2$ and $\lambda_3$. The principal invariants of $U$ and $V$ are

$$I = \lambda_1 + \lambda_2 + \lambda_3$$
$$II = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1$$
$$III = \lambda_1\lambda_2\lambda_3$$

(8)

The Cayley–Hamilton theorem declares that every tensor satisfies its own characteristic equation. That is, for the second order tensor $U$

$$U^3 - IU^2 + IIU - IIII = 0$$

(9)

Some well-known relations of the Seth–Hill strain measures with their conjugate stresses are as follows [6,7]:

(i) Green’s strain and second Piola–Kirchhoff stress tensors:

$$E^{(2)} = \frac{1}{2}(U^2 - I); \quad T^{(2)} = IIIF^{-1}\sigma F^{-T}$$

(10)

(ii) Biot stress and strain tensors [5]:

$$E^{(1)} = U - I; \quad T^{(1)} = \frac{1}{2}(T^{(2)}U + UT^{(2)})$$

(11)

(iii) The conjugate pairs $T^{(-1)}$ and $E^{(-1)}$ [2]:

$$E^{(-1)} = I - U^{-1}; \quad T^{(-1)} = \frac{1}{2}(T^{(-2)}U^{-1} + U^{-1}T^{(-2)})$$

(12)

(iv) Almansi strain and the weighted convected stress tensors:

$$E^{(-2)} = \frac{1}{2}(I - U^{-2}); \quad T^{(-2)} = IIIF^T\sigma F$$

(13)

(v) Logarithmic strain $E^{(0)} = \ln(U)$ and its conjugate $T^{(0)}$ [3]:
Also, as a useful equation, the following equality relates two conjugate stresses $T^{(n)}$ and $T^{(-n)}$ [10]:

$$T^{(-n)} = U^n T^{(n)} U^n$$

where the power of $U$ can be reduced by the Caley–Hamilton theorem.

2. The Hyper-elastic constitutive equation $T^{(n)} = 2\mu E^{(n)} + \lambda tr(E^{(n)})I$

As cited by Hill [7], as a natural generalisation of Hook’s law, Eq. (6) offers a framework for elastic constitutive laws to be explored. It is noted that both $E^{(n)}$ and $T^{(n)}$ are Lagrangian objective [5], that is their components are constant under any rigid body rotations and are the same in all reference frames, hence presenting (6) as an objective equation. In this investigation, uniaxial loading and simple shear tests are used to predict the behaviour of material for finite elastic strains.

2.1. Unaxial loading of a cylinder

Consider a cylinder with circular cross section under axial load $P$ which is uniformly distributed at the end cross sections (Fig. 1).

The Lagrangian description of deformation of the body particles are:

$$x_1 = \lambda_1 X_1; \quad \lambda_1 = \frac{l}{L}$$
$$x_2 = \lambda_2 X_2; \quad \lambda_2 = \frac{r}{R}$$
$$x_3 = \lambda_3 X_3; \quad \lambda_3 = \frac{r}{R}$$

where $L$ and $l$ are initial and final lengths, and $R$ and $r$ are the initial and final radii of the element, respectively. We can obtain:

$$F = U = V = \begin{bmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_2 & 0 \\
0 & 0 & \lambda_2
\end{bmatrix}$$

Fig. 1. The uniaxial test.
and \( R = I \). In this case, the principal and fixed coordinate axes are coincident. Combination of Eqs. (3), (4), and (6) for \( m = 2 \) yields:

\[
\frac{n}{2} \begin{bmatrix} \mathbf{T}^{(2)}_{11} & \mathbf{T}^{(2)}_{12} & \mathbf{T}^{(2)}_{13} \\ \mathbf{T}^{(2)}_{21} & \mathbf{T}^{(2)}_{22} & \mathbf{T}^{(2)}_{23} \\ \mathbf{T}^{(2)}_{31} & \mathbf{T}^{(2)}_{32} & \mathbf{T}^{(2)}_{33} \end{bmatrix}^{\lambda_2^{2-n}} = \frac{2\mu}{n} \begin{bmatrix} \lambda_1^n - 1 & 0 & 0 \\ 0 & \lambda_2^n - 1 & 0 \\ 0 & 0 & \lambda_3^n - 1 \end{bmatrix} + \frac{\lambda}{n} \left( \lambda_1^n + \lambda_2^n + \lambda_3^n - 3 \right) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{17}
\]

where

\[
\lambda = \frac{vE}{(1 + v)(1 - 2v)}; \quad \mu = \frac{E}{2(1 + v)} \tag{18}
\]

Here, \( E \) and \( v \) are elastic modulus and Poisson’s ratio respectively. Solving (17) for \( \mathbf{T}^{(2)}_{ij} \) and noting that \( \lambda_2 = \lambda_3 \), it is concluded that:

\[
\mathbf{T}^{(2)}_{11} = \frac{E\lambda_1^{n-2}}{n(1 - 2v)(1 + v)} \left[ (1 - v)\lambda_1^n + 2v\lambda_2^n - v - 1 \right] \tag{19a}
\]

\[
\mathbf{T}^{(2)}_{22} = \mathbf{T}^{(2)}_{33} = \frac{E\lambda_2^{n-2}}{n(1 - 2v)(1 + v)} \left[ v\lambda_1^n + \lambda_2^n - v - 1 \right] \tag{19b}
\]

other components being zero.

The second Piola–Kirchhoff and Cauchy stress tensors are kinematically related as:

\[
\mathbf{\sigma} = III^{-1} \mathbf{F} \mathbf{T}^{(2)} \mathbf{F}^T \tag{20}
\]

Therefore, the Cauchy stress tensor components for the axially loaded cylinder are:

\[
\sigma_{11} = \frac{E\lambda_1^{n-1}}{\lambda_2^2} \frac{\lambda_1^n + 2v\lambda_2^n - v - 1}{n(1 - 2v)(1 + v)} \tag{21a}
\]

\[
\sigma_{22} = \sigma_{33} = \frac{E\lambda_2^{n-2}}{\lambda_1^2} \frac{\lambda_2^n + \lambda_3^n - v - 1}{n(1 - 2v)(1 + v)} \tag{21b}
\]

other components being zero. From equilibrium it is noted that, \( \sigma_{22} = \sigma_{33} = 0 \).

Eq. (4) can be generalized for \( n = 0 \) and we can write expressions between \( \mathbf{T}^{(0)} \) and \( \mathbf{T}^{(2)} \) for \( \lambda_1 \neq \lambda_2 = \lambda_3 \) as:
\[ T_{ij}^{(0)} = \lambda_i^2 T_{ij}^{(2)}; \quad (i = j) \text{ or } (\lambda_i = \lambda_j) \] (22a)

\[ T_{ij}^{(0)} = \frac{1}{2} \ln \left( \frac{\lambda_i^2 - \lambda_j^2}{\lambda_i} T_{ij}^{(2)}; \quad (i \neq j) \& (\lambda_i \neq \lambda_j) \right) \] (22b)

Therefore, Eq. (6) for \( n = 0 \) results in:

\[ \sigma_{11} = E \frac{(1 - v) \ln(\lambda_1) + 2v \ln(\lambda_2)}{(1 - 2v)(1 + v)} \] (23a)

\[ \sigma_{22} = \sigma_{33} = E \frac{v \ln(\lambda_1) + \ln(\lambda_2)}{(1 - 2v)(1 + v)} \] (23b)

However, from the equilibrium equations, we know that the components of real stress \( \sigma \) at a point of the cylinder are:

\[
[\sigma_{ij}] = \begin{bmatrix}
\frac{P}{\lambda_i^2} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\] (24)

where \( A \) is the initial cross sectional area of the element. Therefore, from (21) and (24), for \( n \neq 0 \) we arrive at:

\[ \lambda_2 = \sqrt[2n]{1 + v - 2v\lambda_1^n} \] (25a)

\[ \frac{P}{EA} = \lambda_1^{n-1} \frac{[(1 - v)\lambda_1^n + 2v\lambda_2^n - v - 1]}{n(1 - 2v)(1 + v)} \] (25b)

For \( n = 0 \), from (23) and (24):

\[ \lambda_2 = \lambda_1^{-v} \] (26a)

\[ \sigma_{11} = E \ln(\lambda_1) \] (26b)

\[ \frac{P}{EA} = \lambda_1^{-2v} \ln(\lambda_1) \] (26c)

In order to draw the force–displacement curve, according to the definition, the longitudinal stretch of the element can be written as:

\[ \lambda_1 = l/L = (A + L)/L \] (27)

where \( A \) is the element elongation. Therefore, Eqs. (25)–(27) will give the \( P–A \) curve for the cylinder.
In order to compare these rate-independent models with an objective rate model, we consider the hypo-elastic constitutive equation

\[
\sigma^o = 2\mu D + \lambda \text{tr}(D) I
\]  

(28)

where \(\sigma^o\) is an objective stress rate and is the rate of change of Cauchy stress measured by an observer with a certain motion and spin. The strain rate tensor \(D\) is the symmetric part of velocity gradient tensor \(L\). The observer spin is chosen to be \(\Omega\) (the spin of \(R\)), such that \(\dot{R} = \Omega R\), and hence \(\sigma^o\) is Green–McInnis rate. However, since in a simple tension all the spins vanish, \(\sigma^o\) is equal to material time derivative of \(\sigma\). Therefore, we have

\[
D = L = \ddot{F}F^{-1} = \begin{bmatrix}
\dot{\lambda}_1/\lambda_1 & 0 & 0 \\
0 & \dot{\lambda}_2/\lambda_2 & 0 \\
0 & 0 & \dot{\lambda}_2/\lambda_2
\end{bmatrix}
\]  

(29)

Substitution of (29) into (28) results in:

\[
\begin{bmatrix}
\sigma_{11} & 0 & 0 \\
0 & \sigma_{22} & 0 \\
0 & 0 & \sigma_{33}
\end{bmatrix} = 2\mu \begin{bmatrix}
\dot{\lambda}_1/\lambda_1 & 0 & 0 \\
0 & \dot{\lambda}_2/\lambda_2 & 0 \\
0 & 0 & \dot{\lambda}_2/\lambda_2
\end{bmatrix} + \lambda \left( \frac{\dot{\lambda}_1}{\lambda_1} + 2\frac{\dot{\lambda}_2}{\lambda_2} \right) \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(30)

The time integration of (33) leads to:

\[
\sigma_{11} = (2\mu + \lambda) \ln(\dot{\lambda}_1) + 2\lambda \ln(\dot{\lambda}_2)
\]  

(31a)

\[
\sigma_{22} = \sigma_{33} = \lambda \ln(\dot{\lambda}_1) + 2(\mu + \lambda) \ln(\dot{\lambda}_2)
\]  

(31b)

which satisfies the initial conditions. Comparison of Eqs. (24) and (31) yields:

\[
\dot{\lambda}_2 = \lambda_1^{-v}
\]  

(32a)

\[
\sigma_{11} = E \ln(\dot{\lambda}_1)
\]  

(32b)

\[
\frac{P}{EA} = \lambda_1^{-2v} \ln(\dot{\lambda}_1)
\]  

(32c)

This completely coincides with Eqs. (26) for logarithmic strain. This is because the principal axes are fixed and \(R = I\), so that \(D = (\ln U)\) [4] as can be seen in (29). Therefore, from (27) and (32) the force–displacement relation for \(n = 0\) will take the form:

\[
\frac{P}{EA} = \left( 1 + \frac{A}{L} \right)^{-2v} \ln \left( 1 + \frac{A}{L} \right)
\]

(33)

The force–displacement curves of the cylindrical element for different indices \(n\), and for the rate constitutive model, are shown in Fig. 2 assuming ratio \(v = 0.25\).
As can be seen, for $n > 1$ the cylinder stiffens in tension and softens in compression. Therefore, in finite elastic deformation problems, the use of Eq. (6) for $n > 1$ may result in softening in compression zones which may not be physically acceptable. For $n < 0$ the element shows tensile softening and compressive stiffening. For $n = 1$ it shows a linear behaviour and acts like a Hook string which obeys the linear Hook’s law in every range of deformation.

Fig. 3 shows the relative volume change $\det(F) = \frac{dv}{dV}$ for $n \geq 0$, where $dV$ and $dv$ are initial and current infinitesimal volumes of a material particle. As it is seen, excessive finite elastic deformations results in a zero volume, except for $n = 0$. For $n < 0$ the relative volume change is quite unreasonable and are ignored.

2.2. The simple shear test

Fig. 4 shows a rectangular body undergoing a simple shear deformation.
The kinematics of the problem is simply defined in a Lagrangian description as

\[ x_1 = X_1 + \gamma X_2 \]
\[ x_2 = X_2 \]
\[ x_3 = X_3 \]  

(34)

The deformation gradient tensor is

\[
F = \begin{bmatrix}
1 & \gamma & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

(35)

There are several methods to compute \( U \) one of which has been proposed by Hoger and Carlson [9]:

\[
C = F^T F 
\]

(36)

\[
U = \beta_1 (\beta_2 I + \beta_3 C - C^2) 
\]

(37)

where

\[
\beta_1 = \frac{1}{(I.II - III)}; \quad \beta_2 = I.III; \quad \beta_3 = I^2 - II 
\]

(38)

Hence, for simple shear test we have

\[
I = II = 1 + \sqrt{4 + \gamma^2}; \quad III = 1 
\]

(39)

\[
U = \frac{1}{\sqrt{\gamma^2 + 4}} \begin{bmatrix}
2 & \gamma & 0 \\
\gamma & \gamma^2 + 2 & 0 \\
0 & 0 & 1 \\
\end{bmatrix} 
\]

(40)

Now, if we use constitutive equation (6) for the material behaviour, from Eqs. (3), (4) and (6) we can find the components of second Piola–Kirchhoff stress and consequently the Cauchy stress.
tensor components from (20). However, the expressions of stress components become quite complicated and too large to be presented here. For instance the Cauchy stress for \( n = 1 \) is

\[
[\sigma_{ij}] = \alpha \begin{bmatrix}
\cos(\beta)^3 + (4\nu - 3)\cos(\beta)^2 - 2\cos(\beta) + 4(1 - \nu) & \text{Sym.} \\
\sin(2\beta)(1 - \nu - \cos(\beta)/2) & \cos(\beta)^2(1 - \cos(\beta)) \\
0 & 0 \\
2\nu \cos(\beta)(1 - \cos(\beta)) & 0
\end{bmatrix}
\]

where

\[
\alpha = \frac{E}{(1 - 2\nu)(1 + \nu) \cos(\beta)^2} \quad \text{and} \quad \gamma = 2\tan(\beta)
\]

The following basis free equalities will be used to obtain the Cauchy stress components:

\[
T^{(1)} = \frac{1}{2}(T^{(2)}U + UT^{(2)})
\]

\[
T^{(-1)} = UT^{(1)}U = \frac{1}{2}(UT^{(2)}U^2 + U^2T^{(2)}U)
\]

\[
T^{(-2)} = U^2T^{(2)}U^2
\]

Also for \( n = 0 \), the components of stress \( T^{(0)} \) in principal axes of \( U \) are related to principal components of other stresses:

\[
T^{(0)}_{ii} = \lambda_i^{(m)}T^{(m)}_{ii}; \quad i = 1, 2, 3
\]

\[
T^{(0)}_{ij} = \frac{1}{m} \frac{\lambda_i^{(m)} - \lambda_j^{(m)}}{\ln(\frac{\lambda_i^{(m)}}{\lambda_j^{(m)}})} T^{(m)}_{ij}; \quad i \neq j
\]

where \( T^{(0)} \) is energetically conjugate to \( \ln U \). Again, in order to compare these models with an objective rate model, we use the rate constitutive equation (28). Since use of Jaumann rate results in fluctuation in stress components [8], Green–McInnis or Zaremba rate of stress is used for the purpose. That is:

\[
\dot{\sigma}^2 = 2\mu\mathbf{D} + \lambda \text{tr}((\mathbf{D}))\mathbf{I}
\]

where \( \dot{\sigma}^2 \) is

\[
\dot{\sigma}^2 = \dot{\sigma} - \Omega \sigma + \sigma \Omega
\]

and \( \Omega \) is the spin of \( \mathbf{R} \) as
\[ \Omega = \mathbf{R}^{-1} = \begin{bmatrix} 0 & \frac{2\dot{\gamma}}{\gamma^2 + 4} & 0 \\ -\frac{2\dot{\gamma}}{\gamma^2 + 4} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \] (47)

For simple shear, Eqs. (45)–(47) result in a system of differential equations which was first solved by Dienes [8] as:

\[ \sigma_{11} = -\sigma_{22} = \frac{2E}{(1 + v)} \left( \cos(2\beta) \ln(\cos(\beta)) + \beta \sin(2\beta) - \sin(\beta)^2 \right) \] (48a)

Fig. 5. Comparison of \( \sigma_{11} \) for different values of \( n \) and rate constitutive equation for \( v = 0.25 \).

Fig. 6. Comparison of \( \sigma_{22} \) for different values of \( n \) and rate constitutive equation for \( v = 0.25 \).
\[ \sigma_{12} = \frac{E}{(1 + \nu)} \cos(2\beta)(2\beta - 2 \tan(2\beta) \ln(\cos(\beta)) - \tan(\beta)) \] (48b)

\[ \sigma_{33} = 0 \] (48c)

Contrary to Eq. (45), constitutive equation (6) gives rise to \( \sigma_{33} \) for \( n \neq 0 \). The effect of using (6) for different values of \( n \) and the rate constitutive model (45) on true stress components are shown in Figs. 5–8 for \( \nu = 0.25 \).

From Fig. 5 it is seen that \( \sigma_{11} \) is symmetric relative to \( \gamma \), and for \( n = -1 \), \( \sigma_{11} \) is bounded to \( -E/(1 - 2\nu)(1 + \nu) \). For \( n < 0 \), \( \sigma_{11} < 0 \) which is physically unacceptable.

Also, from Fig. 6, for \( n = 1 \), \( \sigma_{22} \) is bounded to \( E/(1 - 2\nu)(1 + \nu) \) when \( \gamma \) increases.

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Fig. 7. Comparison of \( \tau_{12} \) for different values of \( n \) and rate constitutive equation for \( \nu = 0.25 \).

Fig. 8. Comparison of \( \sigma_{33} \) for different \( n \)s and \( \nu = 0.25 \).
In Fig. 7, $\tau_{12}$ is anti-symmetric relative to $\gamma$ for all $n$s and coincide for $n$ and $-n$. For all the Cauchy stress components, there is very good agreement between Eq. (6) for $n = 0$, and Eq. (45).

Finally, Fig. 8 reveals that $\sigma_{33}$ is symmetric relative to $\gamma$ while anti-symmetric relative to $n$. For $n = 0$ and the rate constitutive model, $\sigma_{33}$ is zero.

3. Conclusions

The use of hypo-elastic constitutive equations for large strains in numerical applications usually require special considerations, as the strain does not tend to zero upon unloading in some elastic loading–unloading closed cycles. Furthermore, they require objective rate tensors and incrementally objective algorithms for numerical integration. Hyper-elastic constitutive equations, on the other hand, do not require these considerations. However, their behaviour for large elastic strains is important and investigated in this work. One important feature of these equations is that their behaviour may differ in tension and compression and may show unexpected material hardening or softening. The hyper-elastic constitutive equation (6) for the Seth–Hill strains and their conjugate stresses is investigated in this paper. The relation between two Seth–Hill conjugate stress tensors in the principal axes $\{N_i\}$ or in a basis-free form are well established. These relations help to compute the Seth–Hill conjugate stress components in terms of true or Cauchy stress components. The material behaviour in the uniaxial and simple shear tests is then obtained and the results are compared with an objective rate model equation (45). It is shown that the logarithmic strain and its conjugate stress give results closer to that of the rate model. In addition, the use of Biot stress–strain pairs for a bar element in uniaxial test, results in an elastic spring which obeys the Hook’s law even for large deformations with the same behaviour in tension and compression. The behaviours of other equations are shown to be different in tension and compression.

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